BTS –III–11.14–0941 Reg. No. 15113025

A

B. Tech. Degree III Semester Examination November 2014

IT/CS/CE/SE/ME/EE/EC/EB/EI/FT 301 ENGINEERING MATHEMATICS II

(2006 Scheme)

Time: 3 Hours Maximum Marks: 100

PART A

(Answer *ALL* questions)

(8 x 5 = 40)

I. (a) Reduce the following matrix into its normal form $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ and hence find its rank.

(b) Let $V_1 = (1, -1, 0)$ $V_2 = (0, 1, -1)$ $V_3 = (0, 0, 1)$ be the elements of R^3. Show that the set of vectors $\{V_1, V_2, V_3\}$ is linearly independent.

(c) Obtain the half range sine series for the function e^x in $0 < X < 1$.

(d) Solve the integral equation $\int_0^\infty f(x)\cos\lambda x \, dx = e^{-\lambda}$.

(e) Find Laplace transform of the saw toothed wave of period T given by $f(t) = \frac{t}{T},\ 0 < t < T$.

(f) Find the inverse Laplace transform of $\log\left(\frac{s+1}{s-1}\right)$.

(g) Show that $f = 2xy\,i + (x^2 + 2yz)\,j + (y^2 + 1)\,k$ is a conservative field and find its scalar potential.

(h) Prove that grad $\left(\frac{1}{r}\right) = -\frac{\vec{r}}{r^3}$.

PART B

(4 x 15 = 60)

II. (a) Write the vector $V = (1, -2, 5)$ as a linear combination of vectors $V_1 = (1, 1, 1), V_2 = (1, 2, 3), V_3 = (2, -1, 1)$. (6)

(b) For what values of λ and μ do the system of equations $x + y + z = 6,\ x + 2y + 3z = 10,\ x + 2y + \lambda z = \mu$ have (9)

(i) no solution (ii) unique solution (iii) more than one solution

OR

III. (a) Verify Cayley – Hamilton theorem for the matrix (9)

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ Hence compute A^{-1}

(P.T.O.)

(b) Let T be a linear transformation defined by $T\left[\begin{pmatrix}1 & 1\\1 & 1\end{pmatrix}\right]=\begin{pmatrix}1\\2\\3\end{pmatrix}$. (6)

$T\left[\begin{pmatrix}0 & 1\\1 & 1\end{pmatrix}\right]=\begin{pmatrix}1\\-2\\3\end{pmatrix}$, $T\left[\begin{pmatrix}0 & 0\\1 & 1\end{pmatrix}\right]=\begin{pmatrix}1\\-2\\-3\end{pmatrix}$, $T\left[\begin{pmatrix}0 & 0\\0 & 1\end{pmatrix}\right]=\begin{pmatrix}-1\\2\\3\end{pmatrix}$ Find $T\left[\begin{pmatrix}4 & 5\\3 & 8\end{pmatrix}\right]$.

IV. (a) Expand $f(x)=x\sin x, 0<x<2\pi$ as a Fourier series. (8)

(b) Find the Fourier sine transform of $e^{-|x|}$. Hence show that (7)

$\int_0^\infty \frac{x\sin mx}{1+x^2}dx=\frac{\pi e^{-m}}{2}$, $m>0$.

OR

V. (a) Prove that $x^2=\frac{\pi^2}{3}+4\sum_{n=1}^{\infty}\frac{(-1)^n\cos nx}{n^2}$, $-\pi<x<\pi$ hence evaluate (10)

(i) $\frac{1}{1^2}+\frac{1}{2^2}+\frac{1}{3^2}+\frac{1}{4^2}+\ldots\ldots\ldots\ldots=\frac{\pi^2}{6}$

(ii) $\frac{1}{1^2}-\frac{1}{2^2}+\frac{1}{3^2}-\frac{1}{4^2}+\ldots\ldots\ldots\ldots=\frac{\pi^2}{12}$

(b) Using convolution theorem find the inverse Fourier transform of $\frac{1}{12+7i\omega-\omega^2}$ (5)

VI. (a) Find the Laplace transform of the following functions (8)

(i) $t^2\, e^{-2t}\cos t$

(ii) $\frac{e^{-at}-e^{-bt}}{t}$

(b) Solve the equation $y''-3y'+2y=4t+e^{3t}$, when $y(0)=1, y'(0)=-1$. (7)

OR

VII. (a) Using convolution theorem find $L^{-1}\left[\frac{s^2}{(s^2+a^2)(s^2+b^2)}\right]$ (8)

(b) Evaluate $\int_0^\infty \frac{e^{-2t}\sin^2 t}{t}dt$ (7)

VIII. (a) Find the directional derivative of the function $f=x^2-y^2+2z^2$ at the point $p(1, 2, 3)$ in the direction of the line PQ, where Q is the point $(5, 0, 4)$. (5)

(b) Verify Stokes theorem for $\bar{F}=(x^2+y^2)\,i-2xyj$ taken round the rectangle bounded by the lines $x=a,\ x=-a,\ y=0,\ y=b$. (10)

OR

IX. (a) Prove that $\nabla^2(r^n\bar{r})=n(n+3)\,r^{n-2}\bar{r}$. (5)

(b) Verify divergence theorem for $\bar{F}=4xi-2y^2j+z^2k$ (10)

Taken over the region bounded by the cylinder $x^2+y^2=4,\ z=0,\ z=3$.

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