BTS (C) – III – 11.12 – 060 – B

B.Tech Degree III Semester Examination November 2012

IT/CS/EC/CE/ME/SE/EB/EI/EE/FT 301 ENGINEERING MATHEMATICS II
(2006 Scheme)

Time : 3 Hours | Maximum Marks : 100

PART A
(Answer *ALL* questions)

(8 × 5 = 40)

I. (a) Reduce the following matrix into its normal form and hence find its rank.

$$\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -3 & 4 \end{bmatrix}$$

(b) Let $V_1 = (1,-1,0)$; $V_2 = (0,1,-1)$; $V_3 = (0,2,1)$ and $V_4 = (1,0,3)$ be elements of R^3. Show that the set of vectors $\{V_1, V_2, V_3, V_4\}$ is linearly dependent.

(c) Find the Fourier integral of $f(x) = \sin x;\ 0 < x < \pi$.

(d) Find the Fourier sine transform of $\frac{e^{-ax}}{x}$.

(e) Find the Laplace transform of the saw toothed; wave of period T given $f(t) = \frac{t}{T},\ 0 < t < T$.

(f) Apply convolution theorem to evaluate $L^{-1}\left(\frac{1}{(s+a)(s+b)}\right)$.

(g) If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $r = |\vec{r}|$, then prove that $\nabla r^n = nr^{n-2}\vec{r}$.

(h) Find the work done when a force $\vec{F} = (x^2 - y^2 + x)\vec{i} - (2xy + y)\vec{j}$ moves a particle in the XY-plane from (0.0) to $(1,1)$ along the parabola $y^2 = x$.

PART B

(4 × 15 = 60)

II. Verify Cayley-Hamilton theorem for the matrix (15)

$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & 1 \\ 1 & -1 & 2 \end{bmatrix}$. Hence compute A^{-1}. Also find the eigen values and eigen vectors.

OR

III. (a) Solve the simultaneous equations using the help of matrices. (10)

$x + y + z = 3$

$x + 2y + 3z = 4$

$x + 4y + 9z = 6;$

Check whether it is consistent or not.

(b) Let T be a linear transformation defined by (5)

$$T\left[\begin{pmatrix} 1 & 1 \\ 1 & 1 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix};\ T\left[\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix};\ T\left[\begin{pmatrix} 0 & 0 \\ 1 & 1 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ -2 \\ -3 \end{pmatrix}$$

$$T\left[\begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}\right] = \begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}.\quad \text{Find } T\left[\begin{pmatrix} 4 & 5 \\ 3 & 8 \end{pmatrix}\right].$$

IV. (a) Find the Fourier series expansion for $f(x)$: *if* (10)

$$f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x, & 0<x<\pi\end{cases}$$

Deduce that $\frac{1}{1^2}+\frac{1}{3^2}+\frac{1}{5^2}+\frac{1}{7^2}+\ldots\ldots\ldots=\frac{\pi^2}{8}$.

(b) State and prove the linear property of Fourier transforms. (5)

OR

V. (a) Find the Fourier series expansion of the following periodic function of period; (10)

$$f(x)=\begin{cases}2+x & -2\le x\le 0\\ 2-x & 0\le x\le 2\end{cases}$$

(b) Define half range sine series and cosine series. (5)

VI. (a) Find the Laplace transform of (8)

(i) $e^t\cos t$ (ii) $\frac{e^{-at}-e^{-bt}}{t}$ (iii) $\sin t\; U(t-\pi)$

(b) Find the inverse Laplace transform of (7)

(i) $\log\left(\frac{s^2+s}{s^2+4}\right)$ (ii) $\frac{5s^2+3s-16}{(s-1)(s-2)(s+3)}$

OR

VII. (a) Solve the simultaneous equations (10)

$(D^2-3)x-4y=0$

$x+(D^2+1)y=0$ for $t>0$ given that $x=y=\frac{dy}{dt}=0$ and $\frac{dx}{dt}=2$ at $t=0$.

(b) Find the Laplace transform of the periodic function of period 2π; defined by (5)

$$f(t)=\begin{cases}1 & \text{for } 0\le t<\pi\\ -1 & \text{for } \pi<t\le 2\pi\end{cases}$$

VIII. (a) Verify Gauss divergence theorem for $\vec{V}=3x^2\vec{i}+6y^2\vec{j}+z\vec{k}$ for the region bounded by the closed cylinder $x^2+y^2=16$; $z=0$ and $z=4$. (10)

(b) If $\vec{V}=3x^2y^2Z^4\vec{i}+2x^3yz^4\vec{j}+4x^3y^2z^3\vec{k}$, show that $\vec{V}$ is a conservative field. (5)

OR

IX. (a) If $\vec{F}=(2x^2-3z)\vec{i}-2xy\vec{j}-4x\vec{k}$; then evaluate $\iiint_v \nabla.\vec{F}dv$ where V bounded by the planes $x=0$, $y=0$, $z=0$ and $2x+2y+z=4$. (8)

(b) Prove that $curl(curl\vec{f})=grad(div\vec{f})-\nabla^2\vec{f}$. (7)

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