B. Tech Degree III Semester Examination November 2010

IT/CS/EC/CE/ME/SE/EB/EI/EE/FT 301 ENGINEERING MATHEMATICS II

(2006 Scheme)

Time : 3 Hours Maximum Marks : 100

PART – A

(Answer *ALL* questions)

(8 x 5=40)

I. (a) Find the rank of $A=\begin{bmatrix}1 & -1 & 3 & 6\\ 1 & 3 & -3 & -4\\ 5 & 3 & 3 & 11\end{bmatrix}$ by reducing to the normal form.

(b) If x, y, z are linearly independent vectors of a vector space V, prove that $x, x+y, x+y+z$ are also linearly independent.

(c) Find the inverse Fourier transform of $\frac{e^{4i\omega}}{3+i\omega}$.

(d) Find the amplitude representation of the function

$$f(t)=5, \quad -2\le t\le 2$$
$$=0, \quad otherwise$$

(e) Find the Laplace transform of $te^{-t}Sin3t$.

(f) Find the inverse Laplace transform of $\frac{3S+1}{(S+1)^4}$.

(g) Find the directional derivative of the function $2xy+z^2$ in the direction of the vector $\bar{i}+2\bar{j}+2\bar{k}$ at $(1,-1,3)$

(h) If $\bar{A}$ and $\bar{B}$ are irrotational, prove that $\bar{A}\times\bar{B}$ is solenoidal.

PART B

(4 x 15 = 60)

II. (a) Check whether the given system of equations is consistent or not. If consistent solve it

$$2x-y+3z=4$$
$$x+y-3z=-1$$
$$5x-y+3z=7$$

(7)

(b) Let V be the set of all positive real numbers with addition defined by $x+y=xy$ and scalar multiplication defined by $\alpha x=x$. Examine whether V is a vector space. (8)

OR

III (a) If $A=\begin{bmatrix}1 & 0 & 0\\ 1 & 0 & 1\\ 0 & 1 & 0\end{bmatrix}$ show that $A^n=A^{n-2}+A^2-I, n\ge 3$. Hence find A^{50}. (8)

(b) Let V be the set of all vectors of the form (x_1,x_2,x_3) in R^3 satisfying (i) $x_1-3x_2+2x_3=0$ (ii) $3x-2x_2+x_3=0$ and (iii) $4x_1+5x_2=0$. Find the dimension and basis for V. (7)

(P.T.O)

IV. (a) Find a Fourier Series to represent $x - x^2$ from $x = -\pi$ to π. Hence show that

$$\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \dots\dots\dots\dots = \frac{\pi^2}{12}.$$ (8)

(b) If $f(x) = x,\ 0 < x < \frac{\pi}{2}$

$$= \pi - x, \frac{\pi}{2} < x < \pi$$

Show that $f(x) = \frac{4}{\pi}\left[Sinx - \frac{Sin3x}{3^2} + \frac{Sin5x}{5^2} - \dots\dots\dots \right].$ (7)

OR

V. (a) Find the Fourier Sine and Cosine transforms of

$$f(t) = e^{2t} - e^{-2t},\ 1 \le t < 2$$

$$= 0, \quad otherwise$$ (7)

(b) Using Fourier Sine integral, show that

$$\int_0^\infty \frac{1 - Cos\pi\omega}{\omega} Sin\omega x \, d\omega = \begin{cases} \frac{1}{2}\pi, & 0 < x < \pi \\ 0, & x > \pi \end{cases}$$ (8)

VI. (a) Solve $y''' + 2y'' - y' - 2y = 0$, given that $y(0) = y'(0) = 0,\ y''(0) = 6.$ (8)

(b) Find the Laplace transform of the square wave function

$$f(t) = K\ ,\ 0 \le t \le a$$

$$= -K, a \le t \le 2a$$

and $f(t + 2a) = f(t).$ (7)

OR

VII. (a) Find the inverse Laplace transform of

$(i)\ \frac{S+8}{S^2 + 4S + 5}$ $\quad (ii)\ \log\left(\frac{S+1}{S-1}\right).$ (8)

(b) Find the Laplace transform of the unit impulse function. (7)

VIII. (a) A vector field $\vec{V}$ is of the form $\vec{V} = \left(6xy + z^3\right)\vec{i} + \left(3x^2 - z\right)\vec{j} + \left(3xz^2 - y\right)\vec{k}$

show that $\vec{V}$ is a conservative field and find its scalar potential. (8)

(b) Prove that Curl $\left(f\vec{V}\right) = \left(gradf\right) X \vec{V} + f\left(Curl\vec{V}\right).$ (7)

OR

IX. (a) Evaluate $\iint_S \vec{A}.\vec{n}\, ds$, where $\vec{A} = \left(x^2 + y^2\right)\vec{i} - 2x\vec{j} + 2yz\vec{k}$ and S is the surface of the plane $2x + y + 2z = 6$ in the first quadrant. (7)

(b) Verify divergence theorem for $\vec{F} = \left(x^2 - yz\right)\vec{i} + \left(y^2 - zx\right)\vec{j} + \left(z^2 - xy\right)\vec{k}$ taken over the rectangular parallelopiped $0 \le x \le a, 0 \le y \le b, 0 \le z \le c.$ (8)

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