B. Tech Degree III Semester Examination, November 2009

IT/CS/EC/CE/ME/SE/EB/EI/EE/FT 301 ENGINEERING MATHEMATICS II

(2006 Scheme)

Time : 3 Hours

Maximum Marks : 100

PART – A

(Answer all questions)

(8 x 5=40)

I. a. Write the vector V=(2, -5, 3) in R^3 as a linear combination of the vectors e_1=(1, -3, 2), e_2=(2, -4, -1) and $e_3 = (1, -5, 7)$, if possible.

b. Solve the following system of equation by matrix inversion method:

$$3x + y + 2z = 3$$
$$2x - 3y - z = -3$$
$$x + 2y + z = 4$$

c. Expand the function defined by

$$f(x) = 0 \text{ for } -2 < x < 0$$
$$= x \text{ for } 0 < x < 2$$

as a full series in [-2, 2].

d. Find the Fourier sine transform of $e^{-|x|}$

e. Find the Laplace transform of $\frac{1 - Cos2t}{t}$

f. Evaluate $\int_0^\infty te^{-2t}$ Sin t dt using Laplace transforms.

g. Prove that $\nabla^2 \left(r^n \bar{r}\right) = n(n+3)r^{n-2}\bar{r}$.

h. Obtain a, b, c such that the vector $F = (x + y + az)i + (bx + 2y - z)j + (-x + cy + 2z)k$ is irrotational.

PART B

(4 x 15 = 60)

II. a. Show that the equations x + y + z = a, 3x + 4y + 5z = b, 2x +3y + 4z = c

(i) have no solution if a=b=c=1

(ii) have many solutions if $a = \frac{b}{2} = c = 1$. (8)

b. Show that the transformation $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular. Write down the inverse transformation. (7)

OR

III. a. Determine the values of 'k' for which the following set of equation may possess non-trivial solution and find it.

$$3x_1 + x_2 - kx_3 = 0,\ 4x_1 - 2x_2 - 3x_3 = 0,\ 2kx_1 + 4x_2 + kx_3 = 0.$$ (8)

(Turn over)

b. Check whether the set $w = \{(a, b, 2a+3b) : a,b \in R\}$ is a subspace or not. (

c. Examine whether (1,0,1), (1,1,0), (0,1,1) is a basis of R^3. (3

IV a. Find the Fourier series to represent *f(x)* in $[-\pi, \pi]$, given

$$f(x) = \begin{cases} x + x^2, & -\pi < x < \pi \\ \pi^2, & x = \pm\pi \end{cases}$$

Deduce that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \dots\dots\dots = \frac{\pi^2}{6}$. (8)

b. Solve the integral equation: $\int_0^\infty f(x) Cos\alpha x dx = e^{-\alpha}$. (7)

OR

V. a. Find the Fourier cosine transform of e^{-x^2}. (10)

b. If the Fourier sine transform of *f(x)* is $\frac{1 - Cos\, x\pi}{x^2\pi^2}$, $0 < x < \pi$, find $f(x)$. (5)

VI. a. Prove that $\int_0^\infty \frac{e^{-t} Sin^2 t}{t} dt = \frac{1}{4} \log 5$. (8)

b. Find the inverse Laplace transform of $\tan^{-1}\left(\frac{2}{5}\right)$. (7)

OR

VII. a. Use Laplace transform method to solve $y'' - 3y' + 2y = 4t + e^{3t}$ when $y(0) = 1$, $y'(0) = -1$. (8)

b. Find the Laplace transform of $e^{-3t}u(t-2)$. (7)

VIII. a. Using Divergence theorem, calculate $\iint \overline{F}.\overline{x}ds$ where $\overline{F} = 2xz\,i + yz\,j + z^2k$ over the half sphere $x^2 + y^2 + z^2 = a^2$ lying above the IC plane. (6)

b. Verify stokes theorem for the function $\overline{f} = (x^2 + y^2)i - 2xy\,j$ around the rectangle bounded by the lines $x = \pm a$, $y = 0$, $y = b$. (9)

OR

IX. a. Find the directional derivative of $\phi(x, y, z) = xy^2 + yz^3$ $at\,(2, -1, 1)$ in the direction of the vector $i + 2j + 2k$. (5)

b. Evaluate $\oint_c xydx + xy^2dy$ by Stoke's theorem where c is the square in the *xy* plane with vertices (1,0), (-1,0), (0,1), (0,-1). (10)

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