

B.Tech. Degree III Semester Examination, November 2008
IT/CS/EC/CE/ME/SE/EB/EI/EE/FT 301 ENGINEERING MATHEMATICS II
 (2006 Scheme)

Time: 3 Hours

Maximum Marks: 100

(8 x 5 = 40)

PART A

(Answer All questions)

- I
- a) Define the rank of a matrix. Find the rank of A by reducing to the normal form.
- $$A \begin{bmatrix} 1 & -1 & 3 & 6 \\ 1 & 3 & -3 & -4 \\ 5 & 3 & 3 & 11 \end{bmatrix}$$
- b) Let $V = \{f / f \text{ is a real valued continuous function on } [a, b] \text{ such that } f(b) = p, p \neq 0\}$. Then verify whether V is a vector space.
- c) Express $f(x) = |x|, -\pi < x < \pi$ as a Fourier series.
- d) Find the Fourier transform of the function $f(t) = e^{-a|t|}, -\alpha < t < \alpha, a > 0$.
- e) Find the Laplace transform of:
- $u_a(t)$, unit step function
 - $f(t) = t^2 u_3(t)$
- f) Using Laplace transform, solve the initial value problem $y'' + 3y' + 2y = 3$, given $y(0) = 1, y'(0) = 1$.
- g) If $\vec{r} = xi + yj + zk$, and $r = |\vec{r}|$, then prove that $\nabla r^n = nr^{n-2} \vec{r}$.
- h) If $\vec{v} = 3x^2 y^2 z^4 i + 2x^3 yz^4 j + 4x^3 y^2 z^3 k$, show that \vec{v} is a conservative field and find its scalar potential.

PART B

(4 x 15 = 60)

- II
- a) If $A \begin{bmatrix} 1 & 0 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$, then show that $A^n = A^{n-2} + A^2 - I, n \geq 3$. Hence find A^{50} . (10)
- b) Find the dimension of the subspace of R^4 spanned by $\{(1, 0, 0, 0), (0, 1, 0, 0), (1, 2, 0, 1), (0, 0, 0, 1)\}$. Hence find its basis. (5)

OR

- III
- a) Determine the eigen values and eigen vectors of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$. (8)
- b) Let T be a linear transformation from R^2 to R^3 such that $Tx = Ax$, where $A = \begin{bmatrix} 2 & 1 \\ 1 & -1 \\ 3 & 2 \end{bmatrix}$. Find $\ker(T)$, $\text{ran}(T)$ and their dimensions. (7)

(Turn Over)

- IV a) Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$. Hence show that $\frac{1}{1^2} - \frac{1}{2^2} + \frac{1}{3^2} - \frac{1}{4^2} + \dots = \frac{\pi^2}{12}$. (10)
- b) Find the Fourier transform of the function $f(t) = e^{-at^2}$, $a > 0$. (5)
- OR**
- V a) Find the Fourier cosine and sine integral representation of $f(x) = e^{-kx}$, $x \geq 0$, where k is a positive constant. (10)
- b) Using convolution find the inverse Fourier transform of $\frac{1}{12 + 7iw - w^2}$. (5)
- VI a) Find the Laplace transform of
- i) $(\text{Cot} + \text{Sin } t)^2$ ii) $e^t \text{Sin } t$ (7)
- b) Find the inverse Laplace transform of
- i) $\frac{5S^2 + 3S - 16}{(S-1)(S-2)(S+3)}$ ii) $\frac{3}{S^2 + 2S}$ (8)
- OR**
- VII a) Using convolution find the inverse Laplace transform of $\frac{1}{(S^2 + \omega^2)^2}$. (7)
- b) Find the Laplace transform of the periodic function defined by the triangular wave,
- $$f(t) = \frac{t}{a}, 0 \leq t \leq a$$
- $$\frac{za-t}{a}, a \leq t \leq 2a, f(t+2a) = f(t)$$
- (8)
- VIII a) Prove that $\text{Curl}(f\bar{v}) = (\text{grad } f) \times \bar{v} + f(\text{Curl } \bar{v})$, where f is a scalar function and \bar{v} is a vector function. (10)
- b) Find the directional derivative of the scalar function $f = x^2y - y^2z - xyz$ at $(1, -1, 0)$ in the direction of $i - j + 2k$. (5)
- OR**
- IX a) Evaluate the surface integral $\iint_S \bar{F} \cdot \hat{n} \, dA$, where $\bar{F} = 6zi + 6j + 3yk$ and S is the portion of the plane $2x + 3y + 4z = 12$ which is in the first octant. (8)
- b) Let D be the region bounded by the closed cylinder $x^2 + y^2 = 16$, $z = 0$ and $z = 4$. Verify the divergence theorem if $\bar{v} = 3x^2i + 6y^2j + zk$. (7)